

What is claimed is:

1 1. An approximation method for a series expansion of an input function with a finite
2 number of terms N to minimize an approximation error, comprising:

3 expanding the input function in Taylor series up to an $(N-1)$ -th term to obtain a first
4 expansion result;

5 expanding the input function in Taylor series up to an N -th term to obtain a second
6 expansion result;

7 multiplying the first expansion result by a predetermined weight α to obtain a
8 multiplication result;

9 combining the multiplication result and the second expansion result to obtain a combined
10 result; and

11 dividing the combined result by $(\alpha + 1)$.

1 2. The method of claim 1, wherein α is greater than 0 and no greater than 1.

1 3. An approximation method for a series expansion of an input function with a finite
2 number of terms N to minimize an approximation error, comprising:

3 expanding the input function in Taylor series up to an $(N-1)$ -th term to obtain an expansion
4 result;

5 multiplying an N -th term of the expansion result by a predetermined weight value to obtain
6 a multiplication result; and

combining the expansion result and the multiplication result to obtain an approximation function f for the series expansion of the input function.

4. The method of claim 3, wherein the predetermined weight value is $\frac{(-1)^N}{(\alpha + 1)}$, for

$0 < \alpha \leq 1$.

5. The method of claim 4, wherein a value α obtained for a corresponding respective N is selected so as to minimize a maximum of the approximation error.

6. The method of claim 4, wherein the value of α is obtained by:

(a) selecting a minimum input in a given input x area;

(b) calculating the approximation function f for the input with the finite number of terms

N ;

(c) obtaining and storing an error $|E_{N,x}|$ by subtracting the approximation function f from a nominal function value of the input x ;

(d) determining whether the input x has reached a maximum value in the given input x area;

(e) adding a predetermined increment ξ to the input x if the input x has not yet reached the maximum value, and repeating steps (b), (c) and (d);

(f) selecting a maximum error value among all the stored errors of $|E_{N,x}|$ for all inputs when

x has reached a maximum value; and

(g) searching the value α to minimize the maximum error value, and storing the value α as the weight value for a corresponding N.

7. The method of claim 3, wherein the value of α is obtained by:

(a) selecting a minimum input in a given input x area;

(b) calculating the approximation function f for the input with the finite number of terms N;

(c) obtaining and storing an error $|E_{N,x}|$ by subtracting the approximation function f from a nominal function value of the input x;

(d) determining whether the input x has reached a maximum value in the given input x area;

(e) adding a predetermined increment ξ to the input x if the input x has not yet reached the maximum value, and repeating steps (b), (c) and (d);

(f) selecting a maximum error value among all the stored errors of $|E_{N,x}|$ for all inputs when x has reached a maximum value; and

(g) searching the value α to minimize the maximum error value, and storing the value α as the weight value for a corresponding N.

8. An approximation method for a series expansion of an input function with a finite number of terms N to minimize an approximation error, comprising:

dividing a whole input area into several predetermined sub-intervals:

4 expanding the input function in Taylor series up to an (N-1)-th term in each of the sub-
5 intervals to obtain a series expansion for each sub-interval;

6 multiplying an N-th term of the series expansion of the input function with a predetermined
7 first weight for inputs on a left side of a center for said each of the sub-intervals;

8 multiplying an N-th term of the series expansion with a predetermined second weight for
9 inputs on a right side of the center for said each of the sub-intervals; and

10 combining the series expansion and the multiplied N-th term with the predetermined first
11 and second weights to obtain an approximation of the input function in said each of the sub-
12 intervals.

1 9. The method of claim 8, wherein the predetermined first and second weights on the
2 left and right side, respectively, in said each of the sub-intervals are selected to minimize a
3 maximum error between the approximation of the input function with the finite number of terms
4 N and a nominal value of the input function over all inputs in corresponding sub-intervals.

1 10. A method for compensating a carrier frequency offset in an orthogonal frequency
2 division multiplexing (OFDM) system, comprising:

3 estimating the carrier frequency offset $\hat{\epsilon}$ by using a series expansion of an arctangent
4 function $\arctan(x)$;

5 using the estimated carrier frequency offset to obtain a phase rotation value for a first input

sample of $k=1$, wherein $\sin(2\pi\hat{\epsilon})$ and $\cos(2\pi\hat{\epsilon})$ are series-expanded to minimize an approximation error;

using a phase rotation value for a previous input sample including $k=1$ to obtain a phase rotation value for a subsequent input sample; and

compensating the phase rotation values for all input samples.

11. The method of claim 10, wherein the estimated carrier frequency offset $\hat{\epsilon}$ is

represented by $\hat{\epsilon} = \frac{1}{2\pi} \arctan \left\{ \frac{\sum_{i=1}^L \text{Im}(y(-i)y^*(L-i))}{\sum_{i=1}^L \text{Re}(y(-i)y^*(L-i))} \right\}$, where Re and Im represent a real part

and an imaginary part, respectively, of a complex number, $y(i)$ represents an i -th received sample, L is a fast fourier transformation (FFT) size, and $\hat{\epsilon}$ is an estimated and normalized carrier frequency offset of $\Delta f T$.

12. The method of claim 11, wherein the phase rotation value for a k-th sample is calculated by:

$$\text{For } k=1, \cos(\Delta\hat{\omega}T_s) = \sum_{n=0}^N (-1)^n \frac{\Delta\hat{\omega}T_s^{2n}}{(2n)!}$$

$$\sin(\Delta\hat{\omega}T_s) = \sum_{n=0}^N (-1)^n \frac{\Delta\hat{\omega}T_s^{(2n+1)}}{(2n+1)!}$$

$$\begin{aligned} \text{For } k \geq 2, \cos(k\Delta\hat{\omega}T_s) &= \cos((k-1)\Delta\hat{\omega}T_s + \Delta\hat{\omega}T_s) \\ &= \cos((k-1)\Delta\hat{\omega}T_s) \cos(\Delta\hat{\omega}T_s) - \sin((k-1)\Delta\hat{\omega}T_s) \sin(\Delta\hat{\omega}T_s) \\ \sin(k\Delta\hat{\omega}T_s) &= \sin((k-1)\Delta\hat{\omega}T_s + \Delta\hat{\omega}T_s) \\ &= \sin((k-1)\Delta\hat{\omega}T_s) \cos(\Delta\hat{\omega}T_s) + \cos((k-1)\Delta\hat{\omega}T_s) \sin(\Delta\hat{\omega}T_s) \end{aligned}$$

13. The method of claim 10, wherein the phase rotation value for a k-th sample is calculated by:

$$\text{For } k = 1, \cos(\Delta\hat{\omega}T_s) = \sum_{n=0}^N (-1)^n \frac{\Delta\hat{\omega}T_s^{2n}}{(2n)!}$$

$$\sin(\Delta\hat{\omega}T_s) = \sum_{n=0}^N (-1)^n \frac{\Delta\hat{\omega}T_s^{(2n+1)}}{(2n+1)!}$$

$$\begin{aligned} \text{For } k \geq 2, \cos(k\Delta\hat{\omega}T_s) &= \cos((k-1)\Delta\hat{\omega}T_s + \Delta\hat{\omega}T_s) \\ &= \cos((k-1)\Delta\hat{\omega}T_s)\cos(\Delta\hat{\omega}T_s) - \sin((k-1)\Delta\hat{\omega}T_s)\sin(\Delta\hat{\omega}T_s) \\ \sin(k\Delta\hat{\omega}T_s) &= \sin((k-1)\Delta\hat{\omega}T_s + \Delta\hat{\omega}T_s) \\ &= \sin((k-1)\Delta\hat{\omega}T_s)\cos(\Delta\hat{\omega}T_s) + \cos((k-1)\Delta\hat{\omega}T_s)\sin(\Delta\hat{\omega}T_s) \end{aligned}$$

14. An approximation system for a series expansion of an input function with a finite number of terms N to minimize an approximation error, comprising:

an operational processing unit which expands the input function in Taylor series up to an (N-1)-th term to obtain a first expansion result, expands the input function in Taylor series up to an N-th term to obtain a second expansion result, multiplies the first expansion result by a predetermined weight α to obtain a multiplication result, combines the multiplication result and the second expansion result to obtain a combined result, and divides the combined result by $(\alpha+1)$.

1 15. The system of claim 14, wherein α is greater than 0 and no greater than 1.

1 16. The system of claim 14, wherein α obtained for a corresponding respective N is
2 selected so as to minimize a maximum approximation error.

1 17. An approximation system for a series expansion of an input function with a finite
2 number of terms N to minimize an approximation error, comprising:
3 an operational processing unit which expands the input function in Taylor series up to an
4 (N-1)-th term to obtain an expansion result, multiplies an N-th term of the expansion result by a
5 predetermined weight value to obtain a multiplication result, and combines the expansion result
6 and the multiplication result to obtain an approximation function f for the series expansion
7 function.

1 18. The system of claim 17, wherein the predetermined weight value is $\frac{(-1)^N}{(\alpha + 1)}$ for
2 $0 < \alpha \leq 1$.

1 19. The system of claim 18, wherein α obtained for corresponding respective N is
2 selected to minimize a maximum approximation error.

1 20. The system of claim 19, wherein α is obtained by:

2 (a) selecting a minimum input in a given input x area;

3 (b) calculating the approximation function f for the input function with the finite number
4 of terms N

5 (c) obtaining and storing an error $E_{N,x}$ by subtracting approximation function f from a
6 nominal function value of the input x ;

7 (d) determining whether the input x has reached a maximum value in the given input x area,
8 adding a predetermined increment ξ to x when x has not yet reached the maximum value, and
9 repeating steps (b), (c) and (d);

10 (e) selecting a maximum error value among all the stored errors $E_{N,x}$ for all inputs when x
11 has reached a maximum value; and

12 (f) searching α to minimize the maximum error value, and storing α as the weight value
13 for a corresponding N .

1 21. The system of claim 17, wherein α obtained for corresponding respective N is
2 selected to minimize a maximum approximation error.

1 22. An orthogonal frequency division multiplexing (OFDM) system for compensating
2 a carrier frequency offset, comprising:

3 an estimator for estimating the carrier frequency offset $\hat{\epsilon}$ by using a series expansion of
4 a function $\arctan(x)$;

5 a first phase rotation calculator for using the estimated carrier frequency offset to obtain
6 a phase rotation value for a first input sample of $k=1$, wherein $\sin(2\pi\hat{e})$ and $\cos(2\pi\hat{e})$ are
7 series-expanded to minimize an approximation error;
8 a second phase rotation calculator for using a phase rotation value for a previous input
9 sample including $k=1$ to obtain a phase rotation value for a subsequent input sample; and
10 a compensator for compensating the phase rotation values for all input samples.

1 23. The system of claim 22, wherein the estimated carrier frequency offset \hat{e} is

2 represented by $\hat{e} = \frac{1}{2\pi} \arctan \left\{ \frac{\sum_{i=1}^L \text{Im}(y(-i)y^*(L-i))}{\sum_{i=1}^L \text{Re}(y(-i)y^*(L-i))} \right\}$, where Re and Im represent a real part

3 and an imaginary part, respectively, of a complex number, $y(i)$ represents an i -th received sample,
4 L is a fast fourier transformation (FFT) size, and \hat{e} is an estimated and normalized carrier
5 frequency offset of $\Delta f T$.

24. The method of claim 23, wherein the phase rotation value for a k-th sample is
calculated by

$$\text{For } k=1, \cos(\Delta\hat{\omega}T_s) = \sum_{n=0}^N (-1)^n \frac{\Delta\hat{\omega}T_s^{2n}}{(2n)!}$$

$$\sin(\Delta\hat{\omega}T_s) = \sum_{n=0}^N (-1)^n \frac{\Delta\hat{\omega}T_s^{(2n+1)}}{(2n+1)!}$$

$$\begin{aligned} \text{For } k \geq 2, \cos(k\Delta\hat{\omega}T_s) &= \cos((k-1)\Delta\hat{\omega}T_s + \Delta\hat{\omega}T_s) \\ &= \cos((k-1)\Delta\hat{\omega}T_s) \cos(\Delta\hat{\omega}T_s) - \sin((k-1)\Delta\hat{\omega}T_s) \sin(\Delta\hat{\omega}T_s) \\ \sin(k\Delta\hat{\omega}T_s) &= \sin((k-1)\Delta\hat{\omega}T_s + \Delta\hat{\omega}T_s) \\ &= \sin((k-1)\Delta\hat{\omega}T_s) \cos(\Delta\hat{\omega}T_s) + \cos((k-1)\Delta\hat{\omega}T_s) \sin(\Delta\hat{\omega}T_s) \end{aligned}$$

25. The method of claim 22, wherein the phase rotation value for a k-th sample is
calculated by

$$\text{For } k=1, \cos(\Delta\hat{\omega}T_s) = \sum_{n=0}^N (-1)^n \frac{\Delta\hat{\omega}T_s^{2n}}{(2n)!}$$

$$\sin(\Delta\hat{\omega}T_s) = \sum_{n=0}^N (-1)^n \frac{\Delta\hat{\omega}T_s^{(2n+1)}}{(2n+1)!}$$

$$\begin{aligned} \text{For } k \geq 2, \cos(k\Delta\hat{\omega}T_s) &= \cos((k-1)\Delta\hat{\omega}T_s + \Delta\hat{\omega}T_s) \\ &= \cos((k-1)\Delta\hat{\omega}T_s) \cos(\Delta\hat{\omega}T_s) - \sin((k-1)\Delta\hat{\omega}T_s) \sin(\Delta\hat{\omega}T_s) \\ \sin(k\Delta\hat{\omega}T_s) &= \sin((k-1)\Delta\hat{\omega}T_s + \Delta\hat{\omega}T_s) \\ &= \sin((k-1)\Delta\hat{\omega}T_s) \cos(\Delta\hat{\omega}T_s) + \cos((k-1)\Delta\hat{\omega}T_s) \sin(\Delta\hat{\omega}T_s) \end{aligned}$$